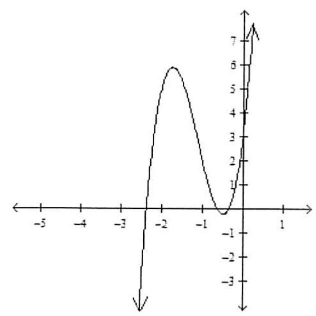
Calculator Permitted Section

		**
1.	C	D
2.	D	B
3.	B	A
4.	C	E
5.	B	A
6.	B	A
7.	D	D

Calculator NOT Permitted Section

		**
8.	A	D
9.	E	C
10.	C	E
11.	D	A
12.	A	D
13.	A	C
14.	D	A

Calculator Permitted Free Response Part A – 2 points total



_____ 1 Draws the graph pictured to the right displaying approximate zeros, correct y – intercept, and correct end behavior.

_____ 1 Since the degree of the function is 3 and the graph displays three roots, then none of the roots of $g(x)$ are imaginary; all are real.

Calculator Permitted Free Response Part B – 2 points total

_____ 1 Possible Rational Roots: $\frac{\text{Factors of 3}}{\text{Factors of 6}} = \frac{\pm1, \pm3}{\pm1, \pm2, \pm3, \pm6} = \pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm3, \pm\frac{3}{2}$

_____ 1 Two Most Probable Rational Roots: $x = -\frac{1}{3}$ and $x = -\frac{1}{6}$ or $-\frac{1}{2}$

Calculator Permitted Free Response Part C – 3 points total

_____ 1 Correct synthetic division for $x = -\frac{1}{3}$ with a remainder of 0

_____ 1 Correct synthetic division for either $x = -\frac{1}{6}$ or $-\frac{1}{2}$ with remainders of $-\frac{17}{18}(-0.944)$ or $-\frac{1}{4}(-0.25)$

_____ 1 Since the remainder was 0 when $x = -\frac{1}{3}$ but the remainder when $x = -\frac{1}{6}$ or $-\frac{1}{2}$ is not 0, then $x = -\frac{1}{3}$ is a rational root but $x = -\frac{1}{6}$ or $-\frac{1}{2}$ is not.

Calculator Permitted Free Response Part D – 2 points total

_____ 1 Applies the quadratic formula to correctly solve $6x^2 + 18x + 9 = 0$, the quadratic that remains after synthetically dividing by the root $x = -\frac{1}{3}$.

$$\frac{6x^2+18x+9}{3} = \frac{0}{3} \Rightarrow 2x^2+6x+3=0 \Rightarrow x = \frac{-6\pm\sqrt{6^2-4(2)(3)}}{2(2)} = \frac{-6\pm\sqrt{12}}{4} = \frac{-6\pm2\sqrt{3}}{4} = \frac{-3\pm\sqrt{3}}{2}$$

_____ 1 Correct roots of $g(x)$: $x = -\frac{1}{3}$, $\frac{-3+\sqrt{3}}{2}$ (or -0.634), and $\frac{-3-\sqrt{3}}{2}$ (or -2.366).

Calculator NOT Permitted Free Response Part A – 2 points total

____ 1 Uses $f(0) = 4$ to show that $c = 4$ $\quad f(0) = a(0)^3 - 11(0)^2 - 8(0) + c = 4 \quad c = 4$
The student may have explained that the constant in the equation represents the y – intercept, which is the point when $x = 0$. Thus, since $f(0) = 4$, then $c = 4$.

____ 1 Uses $f(-1) = 4$ to find that $a = -3$

$$f(x) = ax^3 - 11x^2 - 8x + 4$$
$$f(-1) = a(-1)^3 - 11(-1)^2 - 8(-1) + 4 = 4$$
$$-a - 11 + 8 + 4 = 4$$
$$a = -3$$

Calculator NOT Permitted Free Response Part B – 3 points total

____ 1 As $x \to -\infty$, then $f(x) \to \infty$

____ 1 As $x \to \infty$, then $f(x) \to -\infty$.

____ 1 Justification: The degree of $f(x)$ is odd and the leading coefficient is negative

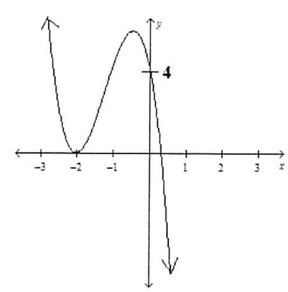
Calculator NOT Permitted Free Response Part C – 2 points total

____ 1 $(x + 2)$ is a factor of $f(x)$ twice.

____ 1 Student shows that when $f(x)$ is synthetically divided by $(x + 2)$ twice, the remainder is zero.

Calculator NOT Permitted Free Response Part D – 2 points total

____ 1 The graph has zeros at $x = -2$ and $x = \frac{1}{3}$ and the graph is tangent to the x – axis at $x = -2$ and passes through the x axis at $x = \frac{1}{3}$ without changing concavity and has a y – intercept of 4.



____ 1 The graph exhibits appropriate end behavior as pictured to the right.

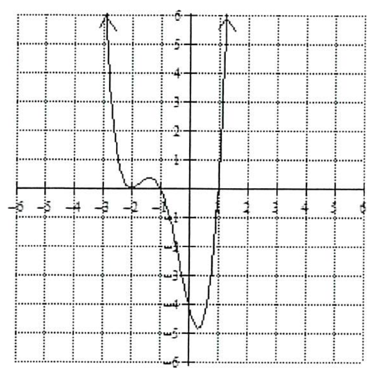
Test #4: Unit #3 – Analysis Polynomial Functions with Irrational and Imaginary Roots

Name____________________________________ Date______________ Period________

Multiple Choice	× (9/7)	
Free Response	× 1	
	Total Score out of 36	

MULTIPLE CHOICE – Calculator Permitted

1. Pictured to the right is the graph of the function $g(x) = ax^4 + 4x^3 + 3x^2 - 4x + b$. Which of the following statements is/are true?



I. The value of $a > 0$.

II. The factor $(x - 2)$ is a factor of $g(x)$ twice.

III. The value of b in the equation is -4.

A. I only　　B. I and II only　　C. I and III only

D. III only　　E. I, II, and III

2. Find all of the roots, real and/or imaginary, of the function $f(x) = x^3 + 6x^2 + 12x + 7$.

A. $x = -1, 7$

B. $x = -1, \frac{-5 \pm \sqrt{3}}{2}$

C. $x = -1, \frac{-5 \pm 3i\sqrt{2}}{2}$

D. $x = -1, \frac{-5 \pm i\sqrt{3}}{2}$

E. Roots cannot be determined

3. Which of the following correctly describes the number of negative roots possible of the function $h(x) = -2x^4 - 3x^3 + 2x^2 - 2x - 3$ according to Descartes' Rule of Signs?

A. 3 or 1　　B. 2 or 0　　C. Only 1

D. 4, 2, or 0　　E. Only 2

4. A quartic function has roots of $x = 1, -3$, and $2i$. What is the equation of $f(x)$?

A. $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$

B. $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

C. $f(x) = x^4 + 2x^3 + x^2 + 8x - 12$

D. $f(x) = x^4 - 2x^3 - 7x^2 + 8x - 12$

E. $f(x) = x^3 - 2x + 3ix - 3$

5. Find the value of k so that the binomial $(x - 3)$ is a factor of the function $f(x) = x^3 + kx^2 + x + 6$.

A. 2

B. -4

C. $\frac{12}{5}$

D. -2

E. None of these

6. For which of the following value(s) of k does the function $g(x) = x^4 - 4x^2 + x + k$ have four distinct real roots?

I. $k = -2$ II. $k = 1$ III. $k = 3$

A. I only B. II only C. I and II only

D. II and III only E. I, II, and III

7. The table of values below represents a cubic polynomial function, $F(x) = ax^3 + 2x^2 - 5x + b$, that has two negative roots and one positive root. Which of the following statements is/are true?

x	-5	-3	-2	-1	0	1	3	5
$F(x)$	-56	0	4	0	-6	-8	24	144

I. The value of $a > 0$ and $b = -6$.

II. In factored form, the equation of $F(x)$ would contain the factor $(x - 3)$.

III. The graph of $F(x)$ passes through the x – axis at $x = -1$ without changing concavity.

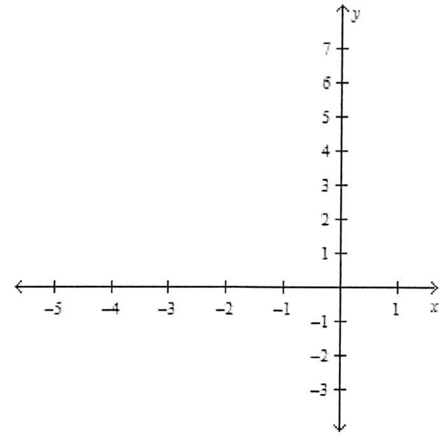
A. I, II and III B. I only C. I and II only

D. I and III only E. II and III only

FREE RESPONSE

Consider the function $g(x) = 6x^3 + 20x^2 + 15x + 3$ to answer the following questions.

a. Use the graphing calculator to sketch a graph of $g(x)$ on the axes to the right. Based on the graph, should any of the roots be imaginary? Give a reason for your answer.



b. Make a complete list of the rational roots that are possible for $g(x)$. Then, after comparing the list to the roots indicated in the graph, choose the two most probable rational roots.

Possible Rational Roots: ______________________________

Two Most Probable Roots: ______________________________

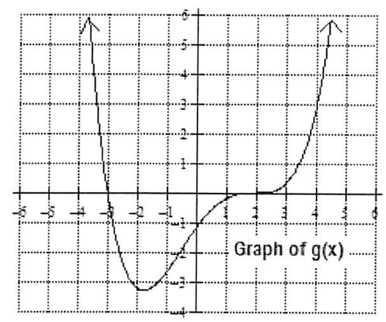
c. Synthetically divide $g(x)$ by both roots that you identified as probable roots in part b. What conclusion can you make from these two synthetic divisions? Give a reason for each of your conclusions.

d. Identify all three roots of $g(x)$. Show your work using the quadratic formula.

The three roots of $g(x)$: ______________________________

MULTIPLE CHOICE – Calculator NOT Permitted Section

8. Which of the following statements is/are true about the quartic function, $g(x)$, pictured to the right.



I. $(x-2)$ is a factor of $g(x)$ a total of 2 times.

II. The equation of $g(x)$ could have had 4 sign changes.

III. If c is the constant term in the equation of $g(x)$, then $c < 0$.

A. III only
B. II only
C. I and II only
D. II and III only
E. I and III only

9. If $x = -3$ is one root of the function $f(x) = x^3 + 5x^2 + 11x + 15$, what are the other two roots?

A. $x = -1$ and -5
B. $x = 1 + 2i$ and $x = 1 - 2i$
C. $x = -1 + i$ and $x = -1 - i$
D. $x = 1$ and -5
E. $x = -1 + 2i$ and $x = -1 - 2i$

10. The synthetic division of a polynomial function, $g(x)$ is shown to the right. Which of the following conclusions can be made?

I. $g(x)$ is a cubic function.
II. The graph of $g(x)$ is below the x – axis at $x = 2$.
III. The graph of $g(x)$ crosses the y – axis at $(0, 8)$.

2	−2	0	3	8
	0	−4	−8	−10
	−2	−4	−5	−2

A. I and III only
B. II and III only
C. I, II, and III
D. II only
E. III only

11. Which of the following is NOT a possible rational root of $g(x) = -6x^3 + 4x^2 - 2x - 2$

A. $-\frac{2}{3}$
B. $-\frac{1}{6}$
C. $\frac{1}{3}$
D. $-\frac{3}{2}$
E. -2

12. Which of the following could be the complete chart of possible types and numbers of the roots of the function $F(x) = -2x^5 + 3x^3 + 2x^2 - x - 3$?

A.

Positive	Negative	Imaginary
2	3	0
2	1	2
0	3	2
0	1	4

B.

Positive	Negative	Imaginary
2	3	0
2	1	2
0	3	2

C.

Positive	Negative	Imaginary
2	3	0
2	1	2

D.

Positive	Negative	Imaginary
2	2	1
2	0	3
0	2	3
0	0	5

13. Which of the following statements is/are true about the function $f(x) = 2x^3 - 4x^2 + 10x - 12$?

I. The graph will fall to the left and rise to the right.

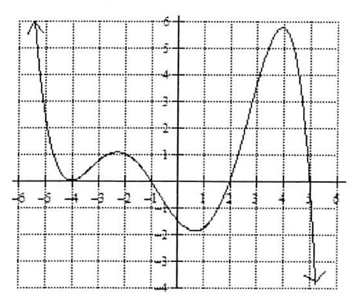
II. There is a guaranteed zero on the interval $1 < x < 2$.

III. One zero of the function is $x = -3$.

A. I and II only B. II and III only C. II only

D. III only E. I, II, and III

14. Assuming that the function graphed below has no imaginary roots, which of the following statements is/are true about the function?



I. The leading coefficient of the equation is positive.

II. The graph of the function has three points of inflection.

III. The function has two roots that are negative, one of which has a multiplicity of 2.

A. I and III only B. III only C. I and II only

D. II and III only E. I, II, and III

FREE RESPONSE

Suppose that $f(x) = ax^3 - 11x^2 - 8x + c$ is such that $f(0) = 4$, and $f(-1) = 4$.

a. Based on the given function values of f, either show or explain why the value of $a = -3$ and the value of $c = 4$.

b. Describe the behavior of the graph of $f(x)$ as $x \to -\infty$ and as $x \to \infty$. Justify your answer.

c. How many times is $(x + 2)$ a factor of $f(x)$? Show the work that leads to your answer.

d. Sketch a possible graph of $f(x)$, correctly labeling all intercepts and displaying correct end behavior.