Test #5: Unit #4 –Analysis of Rational Functions & Inequalities

Name_____ **Date**_____ **Period**_____

Multiple Choice	× (9/7)	
Free Response	× 1	
	Total Score out of 36	

Multiple Choice – Calculator Permitted Section

1. If $P(x)=\frac{(4-3x)(x+3)}{(x+3)(x-2)}$, then which of the following statements is/are true?

I. The graph of $P(x)$ has a horizontal asymptote at $y=-3$.

II. The graph of $P(x)$ has a point discontinuity at $(3,-5)$.

III. The graph of $P(x)$ has a y – intercept at $(0,-2)$.

A. I and II only B. I and III only C. II and III only

D. I only E. I, II, and III

2. Identify the x – value(s) of any removable discontinuity in the function $f(x)=\frac{2x^2-3x-9}{x^2-9}$.

I. $x=3$ II. $x=-3$ III. $x=-\frac{3}{2}$

A. I only B. II only C. I and II only

D. I and III only E. I, II and III

3. A rational function has a vertical asymptote at $x = 2$, a horizontal asymptote at $y = 0$, and a hole in the graph at the point (3, –2). What are the domain and range of the rational function?

A. Domain: $(-\infty,2)\cup(2,3)\cup(3,\infty)$ Range: $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

B. Domain: $(-\infty,2)\cup(2,\infty)$ Range: $(-\infty,\infty)$

C. Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$

D. Domain: $(-\infty,2)\cup(2,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$

E. Domain: $(-\infty,2)\cup(2,3)\cup(3,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$

4. What does the graph of $f(x)=\dfrac{x^2-3x}{x^2+2x-15}$ look like at the value $x = -5$?

A. There is a vertical asymptote at $x = -5$.

B. There is a hole in the graph at $x = -5$.

C. There is a jump in the graph at $x = -5$.

D. The graph is continuous at $x = -5$.

E. The graph has a horizontal asymptote at $x = -5$.

5. Which of the following statements is/are true about the rational function $f(x)=\dfrac{(x-3)(x+1)(x+3)}{(x-2)(x+1)}$?

I. The graph of $f(x)$ has a hole in it at the point $\left(-1,\frac{8}{3}\right)$.

II. The graph of $f(x)$ has a vertical asymptote at $x = 2$.

III. The graph of $f(x)$ has a horizontal asymptote at $y = 3$ and $y = -3$.

A. I only B. I and II only C. II only

D. II and III only E. I, II and III

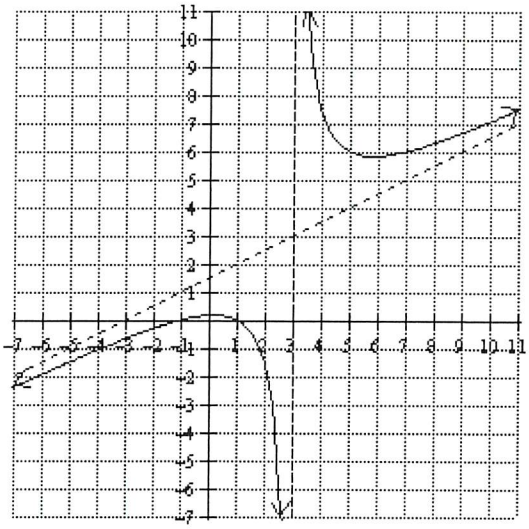
6. Which of the following rational functions have a point discontinuity at $x = 3$ and a horizontal asymptote that is on the x – axis?

I. $f(x)=\dfrac{x^2-x-6}{2x^2-x-15}$ II. $g(x)=\dfrac{3x+9}{x^2-2x-15}$ III. $h(x)=\dfrac{2x-6}{x^2-8x+15}$

A. I and III only B. I and II only C. II only

D. III only E. I only

7. Which of the following statements is/are true about the function, $g(x)$, whose graph is pictured to the right?



I. The factors $(x + 1)$ and $(x - 1)$ are guaranteed to be in the numerator of the equation of $g(x)$.

II. The leading coefficient of the denominator of the equation of $g(x)$ is 2.

III. The factor $(x + 3)$ is guaranteed to be in the denominator of the equation of $g(x)$.

A. I only

B. I and III only

C. I and II only

D. I, II, and III

E. III only

FREE RESPONSE

Pictured below is a table of values that represents a rational function, $H(x)$, whose denominator consists of the factors $(x + 3)$ and $(x - 2)$. Use the information in the table to answer the questions that follow.

x	−1500	−3.001	−3	−2.999	0	1	1.999	2	2.001	1500
$H(x)$	0.999	0.800	Undefined	0.800	0.5	0	−999	Undefined	1001	1.001

a. State the end behavior of the graph of the function $H(x)$. Explain what this behavior says about the leading coefficient of the numerator compared to that of the denominator.

b. According to the table, what can be concluded about the ratio of the constant term of the numerator to the constant term of the denominator? Justify your answer.

c. What factor from the denominator is also in the numerator of the function, $H(x)$? Justify your answer.

d. What factor is in the numerator that is NOT in the denominator? Justify your answer.

Multiple Choice – Calculator NOT Permitted Section

8. The graph of $h(x) = \dfrac{-3x^2 + 3x - 4}{2x - 6}$ …

   A. …has a horizontal asymptote at $y = -\frac{3}{2}$ and a vertical asymptote at $x = 3$.

   B. …has a vertical asymptote at $x = 3$ and a slant asymptote at $y = -\frac{3}{2}x - 3$.

   C. …has a vertical asymptote at $x = -3$ and a slant asymptote at $y = -3x - 6$.

   D. …has a vertical asymptote at $x = 3$ and a slant asymptote at $y = -3x - 6$.

   E. …has a horizontal asymptote at $y = 0$, a vertical asymptote at $x = 3$, and a slant asymptote at $y = -3x - 6$.

9. If it is known that $p(-3) = 0$, which of the following statements is true?

   A. $(x + 3)$ is a non-canceling factor in the numerator.

   B. $(x + 3)$ is a non-canceling factor in the denominator.

   C. The ratio of the constant terms of the numerator and denominator is -3.

   D. $(x - 3)$ is a non-canceling factor in the numerator.

   E. $(x - 3)$ is a non-canceling factor in the denominator.

10. Which of the following statements is true about the function $f(x) = \dfrac{x^2 - 4}{x^2 - 2x - 8}$?

    A. $f(x)$ has two values of x at which point discontinuities exist.

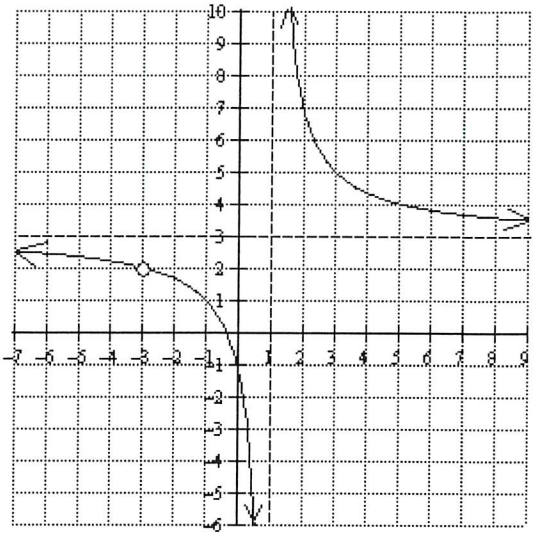
    B. $f(x)$ has one point discontinuity and one infinite discontinuity.

    C. $f(x)$ has one value of x at which a jump discontinuity exists.

    D. $f(x)$ has two values of x at which infinite discontinuities exist.

    E. $f(x)$ is continuous for all values of x.

11. Which of the following rational equations could be the function?

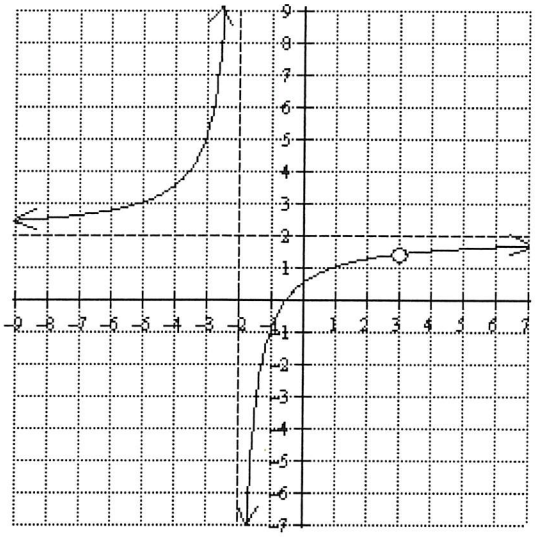


A. $f(x)=\dfrac{(3x+1)(x+3)}{(x-3)(x+3)}$ B. $f(x)=\dfrac{(3x-1)(x-3)}{(x+1)(x-3)}$

C. $f(x)=\dfrac{(3x+1)(x+3)}{(x-1)(x+3)}$ D. $f(x)=\dfrac{(3x-1)(x+3)}{(x+3)(x-1)}$

E. $f(x)=\dfrac{x(x-3)}{(x-3)(x-2)}$

For questions 12 – 13, refer to the graph of the rational function pictured to the right.



12. Which of the following factors is/are in the denominator of the equation of $f(x)$?

I. $(x-2)$ II. $(x+2)$ III. $(x-3)$

A. I only
B. II only
C. I and II only
D. III only
E. II and III only

13. Which of the following statements is/are true about the function, $f(x)$?

I. The degree of the numerator is greater than the degree of the denominator in the equation of $f(x)$.

II. The factor $(x-3)$ appears in both the numerator and the denominator of the equation of $f(x)$.

III. If a and b are the leading coefficients of the numerator and denominator, respectively, then the value of $\frac{a}{b}=2$.

A. II only B. I and II only C. III only

D. II and III only E. I, II, and III

14. If $h(x) = \dfrac{(2-3x)(x+2)}{(x+2)(x-3)}$, then what is the equation of the horizontal asymptote, if one exists?

A. $y = 2$ B. $y = 3$ C. $y = -\frac{2}{3}$

D. $y = -3$ E. No horizontal asymptote exists

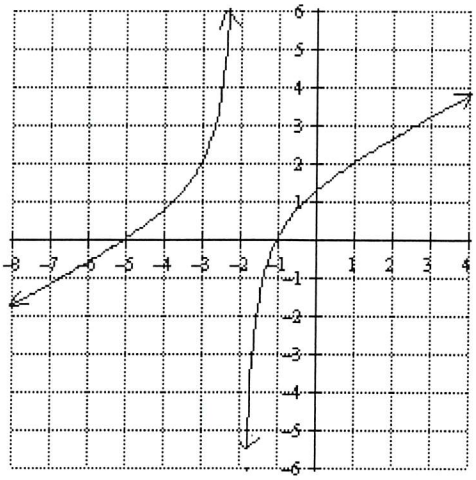
FREE RESPONSE

Consider the rational functions $F(x) = \dfrac{2x^2 - 5x - 3}{(ax+2)(3x-1)}$ and $G(x) = \dfrac{x^2 + 6x + 5}{2x+4}$ to answer the following questions.

a. For what value of a will the graph of $F(x)$ have a horizontal asymptote of $y = \frac{1}{3}$? Show and justify your work.

b. For what value(s) of x is $G(x) \leq 0$. Show the complete sign analysis that leads to your answer.

c. Find the equations of all asymptotes of $G(x)$. Give a reason and show your work to justify their existence and then, graph them on the graph provided below.



Graph of G(x)