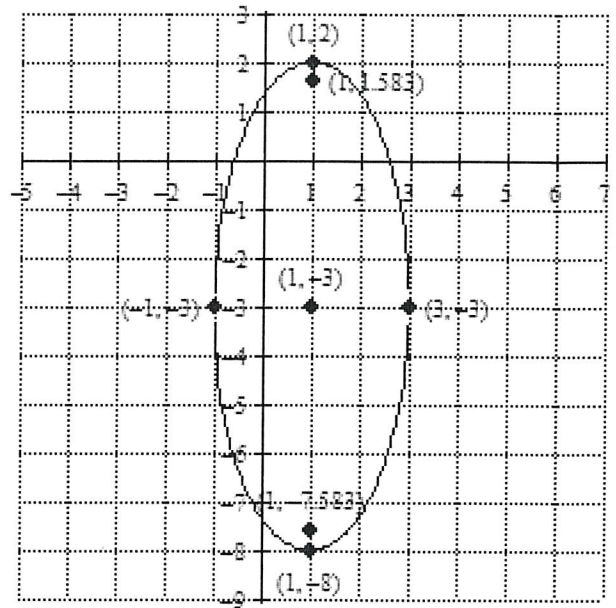# Ellipses

Pictured to the right is the graph of an ellipse. Before we begin to study the relationship between the equation and the graph of an ellipse, we need to first have a vocabulary lesson. Below you will find statements with words underlined. Based on the graph, see if you can come up with a definition of each underlined term. Obviously, the center is the point $(1, -3)$.

1. The points $(1, 2)$ and $(1, -8)$ are the **vertices** of the ellipse.

2. The points $(-1, -3)$ and $(3, -3)$ are the **co-vertices** of the ellipse.

3. The **length of the major axis** is 10.

4. The **length of the minor axis** is 4.

There are two other points labeled on the graph, $(1, 1.583)$ and $(1, -7.583)$. These two points are called the **foci** of the ellipse. From this information, it is a bit far-fetched to expect you to be able to define what these are so let us to a little investigation.

Measure the distance from the top focus to one of the co-vertices. This distance should be the same as what other distance?

Now, construct a right triangle so that one leg of the triangle is the distance from the center to a focus, the other leg is the distance from the center to a co-vertex, and the hypotenuse is the distance from the center to a vertex. Based on the picture you have just drawn, what relationship must be true about the right triangle based on the Pythagorean Theorem?

Study the equations and the graphs of the ellipses below and see if you can determine the connection between them.

$$\frac{(x+2)^2}{4}+\frac{(y-3)^2}{16}=1$$	$$2(x-3)^2+4(y+1)^2=100$$
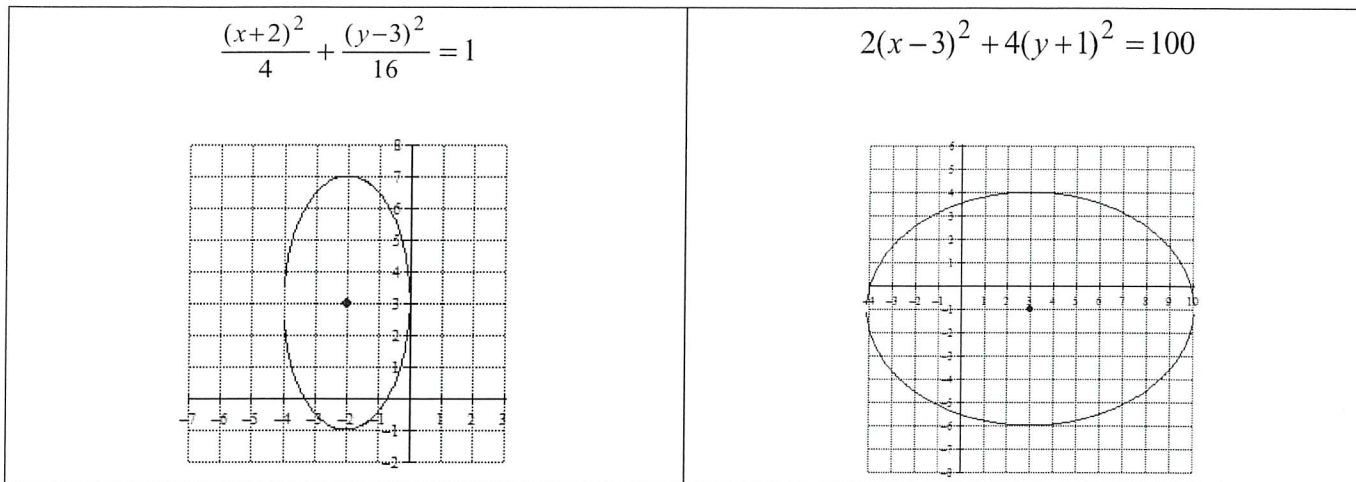
How is the graph of an ellipse different from the graph of a circle? How is this difference noted in the equation of an ellipse versus the equation of a circle?

Identify the following characteristics of the graph of $\frac{(x+2)^2}{4}+\frac{(y-3)^2}{16}=1$ pictured below.

| Coordinates of the Vertices:	Coordinates of the Co-vertices:
Length of the Minor Axis:	Length of the Major Axis:
Coordinates of the Foci:	
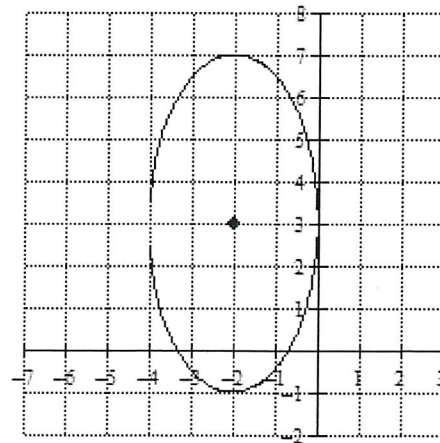
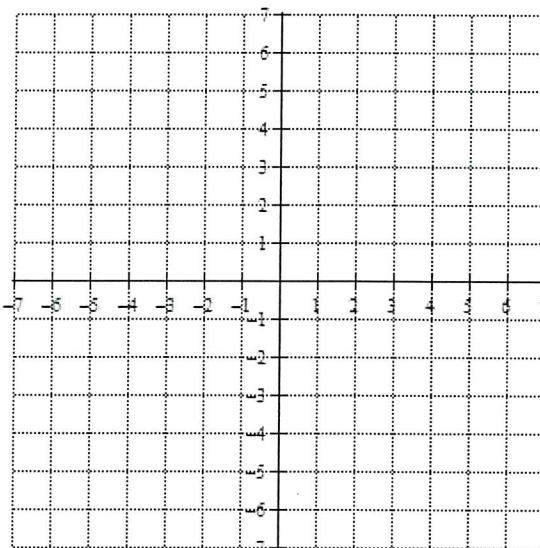Find the equation of each ellipse described below.

The vertices of the ellipse are (2, 0) and (−2, 0) and the length of the minor axis is 2	The center of the ellipse is (2, 3), a vertex is (2, 7) and the coordinates of a focus are (2, 6)
The vertices are (−6, 3) and (2, 3) and the co-vertices are (−2, 5) and (−2, 1)	The center is (4, −3), a vertex is (1, −3), and a focus is $\left(4-\sqrt{2},-3\right)$

Graph the ellipse defined by each equation below. Identify and label the coordinates of the foci for each ellipse.
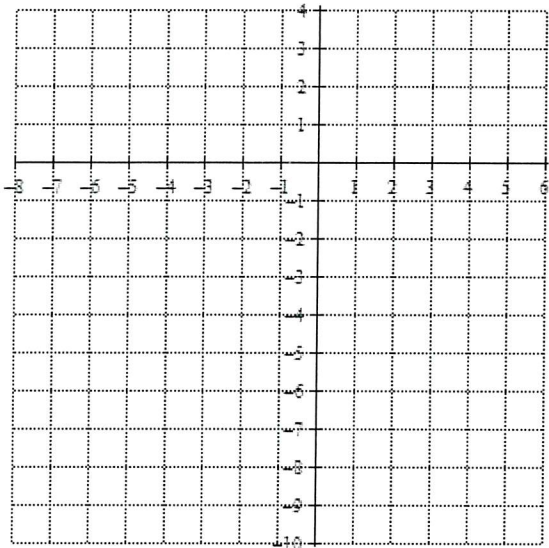
$$\frac{(x+2)^2}{9}+\frac{(y+3)^2}{25}=1$$	$$\frac{(x-1)^2}{16}+\frac{y^2}{9}=1$$

$$4x^2+9y^2+16x-36y+16=0$$