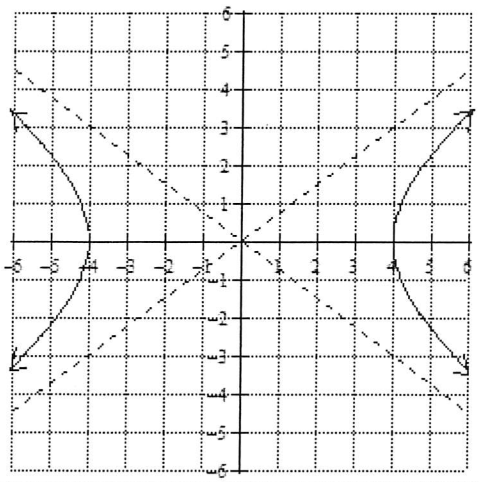
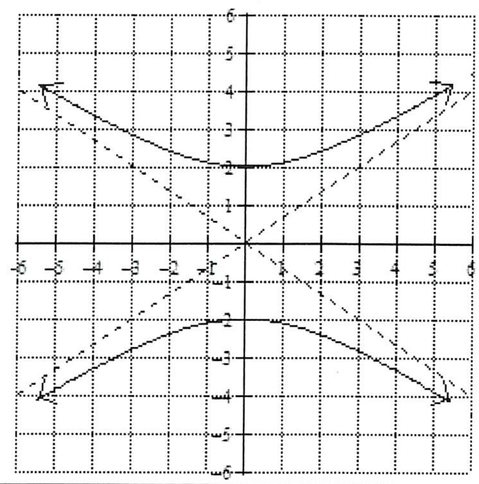
Hyperbolas

Given below are the graphs and equations of two hyperbolas. Study the equations and graphs and answer the questions that follow. But, first, what is the main difference you notice between the equation of an ellipse and the equations of the hyperbolas below?

<table>
<tr><td>$\frac{x^2}{16}-\frac{y^2}{9}=1$
</td><td>$\frac{y^2}{4}-\frac{x^2}{9}=1$
</td></tr>
<tr><td colspan="2">Which variable has a positive coefficient and which variable has a negative coefficient?</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">The transverse axis is defined to be the axis that the branches of the graph cross. For each hyperbola, identify the transverse axis.</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">What is the closest point on each branch of the graph to the intersection point of the asymptotes? These points are called the vertices of the branches.</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">What is the slope of each of the slant asymptotes of each graph?</td></tr>
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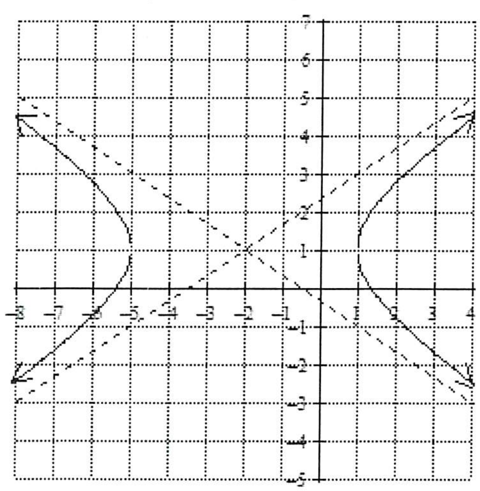
Each of the above characteristics must be determine by something in the equation. Study the equations and your answers to the above questions and at the top of the next page, make three different inferences connecting the equations of hyperbolas to their graphs.

If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$, then answer the following questions about characteristics of the graphs of the hyperbolas.

From the equation, how do you determine which axis is the transverse axis?
From the equation, how do you determine slope of each slant asymptote?
From the equation, how do you determine the intercepts of the transverse axis?

Each of the hyperbolas at the beginning of the lesson were centered at the origin. Now, let's take a look at a hyperbola not centered at the origin and its equation.

$$\frac{(x+2)^2}{9}-\frac{(y-1)^2}{4}=1$$



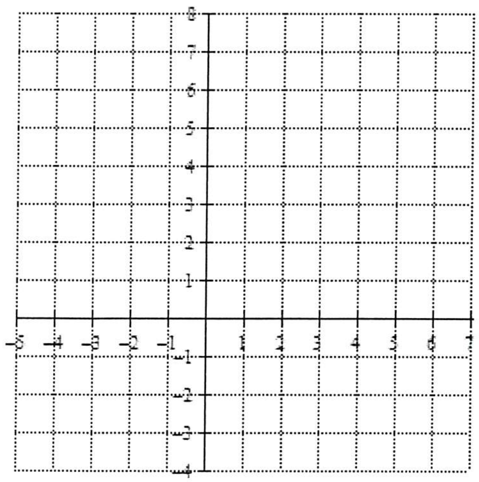
What is the intersection of the asymptotes? How is this determined by the equation provided?

What are the coordinates of the vertices of the branches of the hyperbola? Why do you suppose they are located where they are?

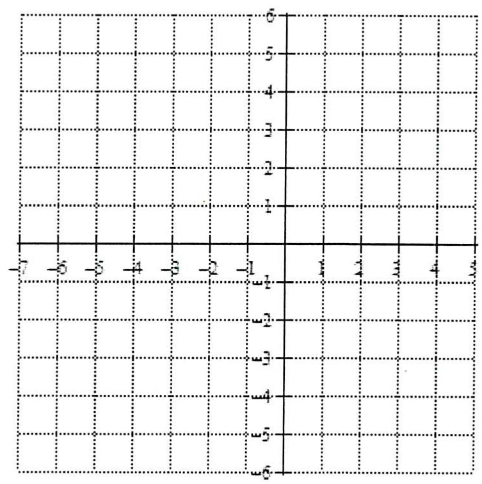
What are the slopes of the asymptotes of the graph? Then, find the equations of the asymptotes of the graph?

Sketch a graph of each of the hyperbolas represented by the implicitly defined equations below. Then, find the equations of the slant asymptotes of each graph.

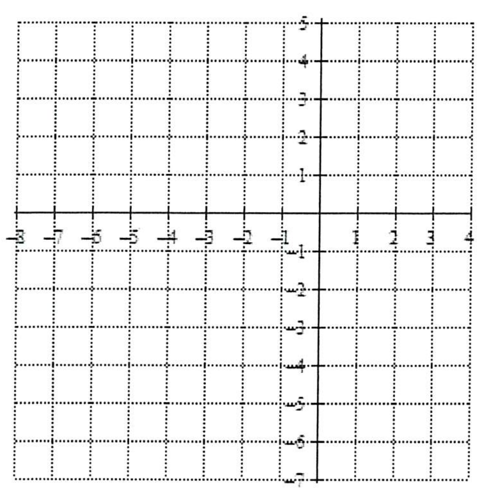
$$\frac{(x-1)^2}{4}-\frac{(y-3)^2}{4}=1$$



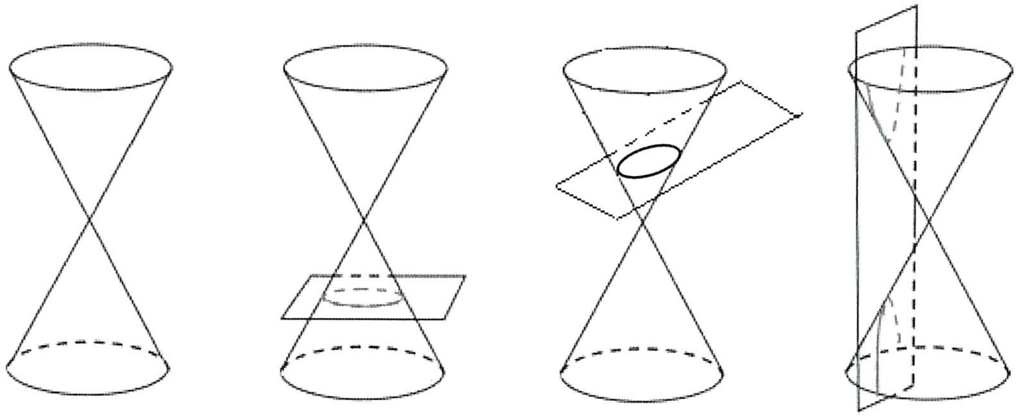
$$\frac{(y-1)^2}{9}-\frac{(x+1)^2}{16}=1$$



$$9x^2-4y^2+36x-8y-4=0$$



Circles, ellipses and hyperbolas are all called conic sections because they are all formed by slicing cones in such a way that faces are created by the cross sections of a plane slicing two cones that are stacked vertex-to-vertex.



If a plane intersects a single cone parallel to the base of the cones, then a(n) ____________ is formed.

If a plane intersects a single cone at an angle to the base of the cones, then a(n) ____________ is formed.

If a plane intersects both cones then a(n) ____________ is formed.

Given the equation of a conic section, identify if the graph will form a circle, an ellipse, or a hyperbola. Give a reason for your answer after putting the equation in standard form.

$x^2+y^2+4x+6y-3=0$	$4x^2+y^2+16x+6y+9=0$	$4x^2-y^2+16x-6y-9=0$

Now, explain how you would be able to tell if the implicitly defined equation, when in standard form, would result in a circle, an ellipse or a hyperbola.