Test #9: Unit #9 – Analysis of Trigonometric Functions

Name_____Date_____Period_____

Multiple Choice	× (9/7)	
Free Response	× 1	
	Total Score out of 36	

MULTIPLE CHOICE – Calculator Permitted

For questions 1 – 3, match the given expressions from the list of trigonometric expressions below that completes each equation to form an identity. If the given expression does not equal any of the choices below, then choose letter E.

A. $\dfrac{1}{1-\cos x}$ B. $\dfrac{1}{\sec^2 x}$ C. $-\dfrac{1}{\csc x}$ D. $\tan x$

1. $-\tan x \cos x = $ _____

2. $\dfrac{\sec x}{\csc x} = $ _____

3. $\dfrac{1+\cos x}{\sin^2 x} = $ _____

4. Which of the following values is/are exact solutions for the equation $2\cos^2 x + 3\cos x + 1 = 0$ on the interval $[0,2\pi)$.

        I. $\dfrac{7\pi}{6}$ II. π III. $\dfrac{2\pi}{3}$

    A. I only

    B. I and II only

    C. I and III only

    D. II only

    E. II and III only

5. Suppose that $\cos\alpha = \frac{4}{5}$ and α terminates in quadrant IV. What is the value of $\tan\left(\alpha - \frac{3\pi}{4}\right)$?

    A. -7

    B. -1

    C. 7

    D. $\frac{1}{7}$

    E. Undefined

6. Which of the following expressions is equivalent to $\sin(\pi + \theta)$?

    A. $\sin\theta$ B. $-\cos\theta$

    C. $-\sin\theta$ D. $\cos\theta$

    E. $\sin\theta - 1$

7. Given that $\cos\theta = -\frac{4}{7}$ and $\pi < \theta < \frac{3\pi}{2}$, what is the value of $\cos 2\theta$?

    A. $-\frac{17}{49}$

    B. $\frac{\sqrt{33}}{7}$

    C. $-\frac{4\sqrt{33}}{14}$

    D. $-\frac{33}{49}$

    E. $\frac{17}{49}$

Let $f(\theta) = \sin 2\theta$ and $g(\theta) = -2\cos^2 \theta$. Use the function to answer the following questions.

a. For what value(s) of θ is $f(\theta) = g(\theta)$ on the interval $0 < \theta \le \pi$. Show your work and leave your answers exact form.

b. On the grid provided below, sketch a graph of both $f(\theta)$ and $g(\theta)$ on the interval $0 < \theta \le \pi$. Then, to the nearest thousandth, find the point(s) at which the two graphs intersect. Explain how this validates or discredits your work from part a).
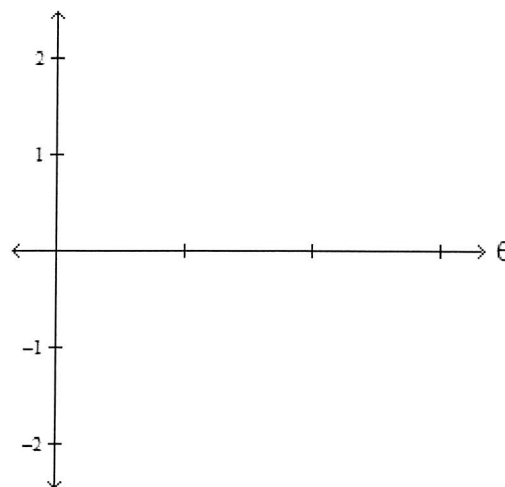
c. For what exact value(s) of θ is $2f(\theta) + 1 = 0$ on the interval $0 < \theta \le 2\pi$. Show your work.

MULTIPLE CHOICE – Calculator NOT Permitted

8. Find all of the solutions on the interval $0 < x \le 2\pi$ for $2\cos x - \sqrt{3} = 0$.

   A. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

   B. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

   C. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

   D. $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

   E. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

9. Simplify the expression $\dfrac{\cos\left(\frac{\pi}{2} - x\right) - \cos\frac{\pi}{2}}{x}$.

   A. $\dfrac{\sin x}{x}$

   B. $\dfrac{\sin x - 1}{x}$

   C. 1

   D. $\dfrac{\cos x - 1}{x}$

   E. $\dfrac{\cos x + 1}{x}$

10. Given that $\cos\left(\frac{\pi}{2} - x\right) = \frac{2}{7}$, what is the value of $\sin^2 x$?

   A. $\dfrac{45}{49}$

   B. $\dfrac{4}{49}$

   C. $\dfrac{45}{4}$

   D. $\dfrac{49}{4}$

   E. $\dfrac{2}{7}$

11. The expression $\dfrac{\csc\theta}{\tan\theta + \cot\theta}$ can be simplified to which of the following expressions?

   A. $\cos\theta + \tan\theta$

   B. $\sin^2\theta + \cos\theta$

   C. $\csc^2\theta\sec\theta$

   D. $\sin\theta$

   E. $\cos\theta$

12. If $\tan \theta < 0$ and $\sin \theta > 0$, determine the quadrant in which θ lies.

    A. I   B. II   C. III   D. IV   E. II or IV

13. Determine the exact value of the expression $\cos \frac{4\pi}{9} \cos \frac{\pi}{9} + \sin \frac{4\pi}{9} \sin \frac{\pi}{9}$.

    A. 1   B. $\dfrac{\sqrt{3}}{2}$ C. -1

    D. 0   E. $\dfrac{1}{2}$

14. Determine which of the following equations represents a trigonometric identity.

    I. $\csc \theta \cos \theta = \cot \theta$ II. $\cos \theta (\tan \theta + \cot \theta) = \csc \theta$ III. $\sin \theta \tan \theta = \sec \theta - \cos \theta$

    A. I and III only   B. II and III only   C. I only

    D. I and II only   E. I, II and III

Let $f(\theta) = \dfrac{\cos(\theta+\pi)}{\sin(\theta-\pi)}$ and $g(\theta) = \dfrac{\cos\theta + \cos^2\theta}{\sin\theta + \sin\theta\cos\theta}$ to answer the following questions.

a. Find the value of $f(\pi)$. Based on your result, what can be concluded about the graph of $f(\theta)$ when $\theta = \pi$? Give a reason for your answer.

b. Analytically show that $f(\theta) = g(\theta)$. Show each step of your analysis.

c. If $f(\theta) = -\frac{3}{5}$, then what is the value of $\tan^2 \theta$. Show your work.