Unit #1 – Limits and Continuity

Name_____**Date**_____

Multiple Choice	× (9/7)	
Free Response	× 1	
	Your Score out of 18	

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

MULTIPLE CHOICE

1. If $x \neq a$, then $\lim_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is…

A. $\dfrac{1}{a^2}$

B. $\dfrac{1}{2a^2}$

C. $\dfrac{1}{6a^2}$

D. 0

E. nonexistent

2. Consider the function $g(x) = \dfrac{x + \cos x}{x + 3}$. Which of the following statement(s) is/are true?

I. $\lim\limits_{x \to -3} g(x)$ does not exist because $\lim\limits_{x \to -3^+} g(x) = \infty$.

II. The graph of $g(x)$ has a horizontal asymptote at $y = 1$ because $\lim\limits_{x \to \infty} g(x) = 1$

III. The intermediate value theorem guarantees a value of c for $g(x)$ on the interval $[-4, 0]$ if $g(c) = -1$.

A. I and II only
B. II and III only
C. I only
D. II only
E. I, II, and III

---

3. If $f(x) = 2x^2 + 1$, then find $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x^2}$.

A. 2
B. Does not exist
C. 1
D. 0
E. ∞

---

4. Find $\lim\limits_{x \to \infty} \dfrac{3x^2 - 2x^5 - x}{x^2 - 3x^5}$.

A. $\frac{2}{3}$
B. ∞
C. 0
D. $-\frac{2}{3}$
E. 3

5. If $\lim_{x\to a} f(x) = -2$ and $\lim_{x\to a} g(x) = 2$, what is the value of $\lim_{x\to a} [3f(x) + g^2(x)]$?

A. 0

B. –2

C. –4

D. 2

E. 8

---

6. Let $f(x) = \begin{cases} x+2 & x<3 \\ 4x-7 & x>3 \end{cases}$ be a function. Which of the following statements are true about f ?

I. $\lim_{x\to 3} f(x)$ exists II. f is continuous at $x = 3$ III. $\lim_{x\to 1} f(x) = 3$

A. I only
B. I and II only
C. I and III only
D. II only
E. III only

---

7. The graph of a function, $g(x)$, is pictured to the right. Find $\lim_{x\to 0^-} \frac{5}{2g(x)}$.

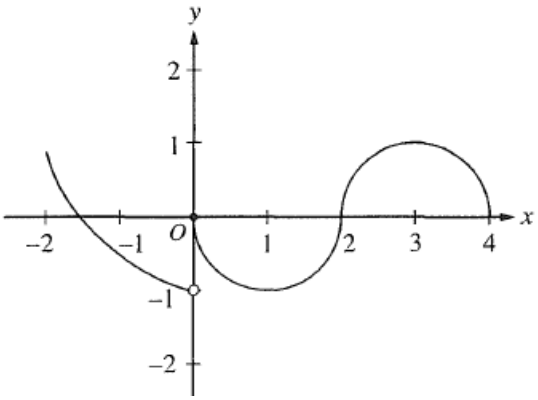
A. $\frac{5}{2}$

B. Does not exist

C. $-\frac{1}{2}$

D. 0

E. $-\frac{5}{2}$

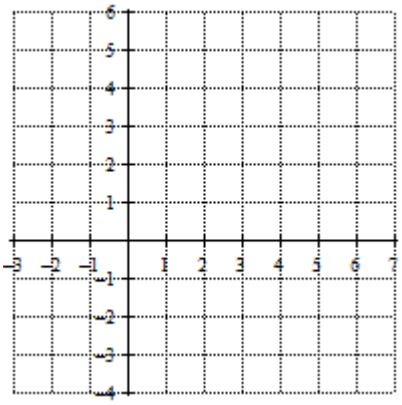


FREE RESPONSE

In order for a function, $f(x)$, to be continuous at a value, $x = c$, three conditions must be true about the function. Keep these conditions in mind as you answer the following questions.

a. Draw a graph of a function that satisfies the following conditions. Then, state if the function is continuous at $x = 3$ or not. If it is not, give the reason(s), based on the three part definition of continuity, why it is not continuous.

$h(3) = 4$ $\quad\quad$ $\lim_{x\to 3^-} h(x) = -2$ $\quad\quad$ $\lim_{x\to 3^+} h(x) = -2$



b. Is the function $f(x) = \begin{cases} 3+\sqrt{2+x}, & x \le 7 \\ 8-\sqrt{x-3}, & x > 7 \end{cases}$ continuous at $x = 7$? Justify your reasoning based on the three part definition of continuity.

c. For what values of k and m will the function $g(x) = \begin{cases} k\sin x + x^2, & 0 \le x < \frac{\pi}{4} \\ x + 2, & \frac{\pi}{4} \le x \le e \\ m\ln e^x, & e < x \le 2\pi \end{cases}$ be continuous on the interval $(0, 2\pi)$? Use limits to justify your work. Round your answers to the nearest thousandth.

Unit #1 – Limits and Continuity

Name_____**Date**_____

Multiple Choice	**× (9/7)**	
Free Response	**× 1**	
	Your Score out of 18	

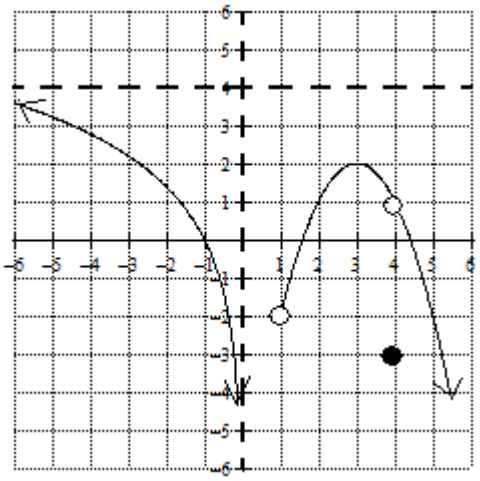
A GRAPHING CALCULATOR IS NOT ALLOWED FOR THIS SECTION OF THE EXAM.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

MULTIPLE CHOICE

Use the graph of $F(x)$ pictured to the right to answer questions 8 – 9.



8. At which of the following values of c does $\lim_{x \to c} F(x)$ NOT exist?

I. $c = 0$ II. $c = 1$ III. $c = 4$

A. I only
B. II only
C. III only
D. I and II only
E. I, II, and III

9. $F(x)$ is discontinuous at $x = 4$ for which of the following reasons?

I. $F(4)$ is undefined II. $\lim_{x\to 4} F(x)$ does not exist. III. $\lim_{x\to 4} F(x) \neq F(4)$

A. I and III only
B. II only
C. II and III only
D. III only
E. I, II, and III

---

10. If $f(x) = \begin{cases} x^2 \cos(\pi x), & 0 < x \le 2 \\ \log_2(x+14), & 2 < x \le 4 \end{cases}$, then the value of $\lim_{x\to 2} f(x)$ is…

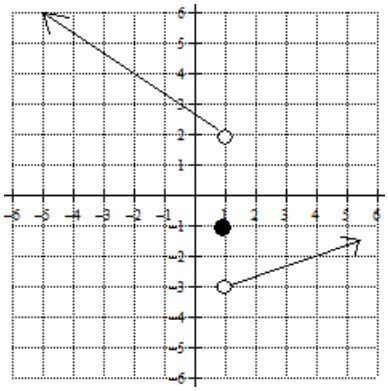
A. 4
B. 2
C. 0
D. –4
E. nonexistent

---

11. Find $\lim_{x\to 4} \dfrac{3-\sqrt{x+5}}{x^2-16}$.

A. $-\frac{1}{48}$
B. ∞
C. 0
D. $\frac{1}{6}$
E. $\frac{1}{72}$

12. The graph of a function, $g(x)$ is pictured to the right. Find the value of $\lim_{x\to 1^+}[2+2g(x)]$.



A. –3
B. –1
C. –4
D. 6
E. Does not exist

13. Find $\lim_{x\to 3^+}\dfrac{2x^2+7x+3}{x^2-9}$.

A. 0
B. ∞
C. $-\infty$
D. ½
E. 2

14. Find $\lim_{x\to 0}\dfrac{\cos 4x\tan 4x}{6x}$.

A. 1
B. 0
C. ∞
D. $\frac{2}{3}$
E. Does not exist

FREE RESPONSE

Consider the rational function $f(x)=\dfrac{2x^2+7x+3}{x^2-x-12}$ to answer the following questions.

a. Find $\lim\limits_{x\to -1}\left[f(x)\cdot 2\cos \pi x\right]$. Show your analysis applying the properties of limits.

b. Does $f(x)$ have any vertical asymptotes? If so, what are they? Show your analysis, justifying your answer using limits.

c. Does $f(x)$ have any horizontal asymptotes? If so, what are they? Show your analysis, justifying your answer using limits.

d. Find $\lim_{x\to\infty} \frac{3x^2+2x}{\sqrt{x^2-2x}}$. What does this result say about the existence of horizontally asymptotic behavior as $x \to \infty$? Show your work and justify your reasoning.