## Characteristics of Quadratic Functions (pp. 1 of 5)

Here are the two forms in which quadratic functions can be written:

| Vertex Form: | $y=a(x-h)^{2}+k$ |
| :--- | :--- |
| General (Standard) Form: | $y=a x^{2}+b x+c$ |

In these equations, $a, b$, and $c, h$, and $k$ represent constants, but a cannot equal zero.

- Why must we say $a \neq 0$ ?


## Comparison of Characteristics

| Characteristic | Standard Form $y=a x^{2}+b x+c$ | Vertex Form $y=a(x-h)^{2}+k$ |
| :---: | :---: | :---: |
| vertex | $\left(\frac{-b}{2 a}, ?\right)$ <br> Plug $x=\frac{-b}{2 a}$ into the equation to find $y$ | (h, k) |
| axis of symmetry | $x=\frac{-b}{2 a}$ | $x=h$ |
| $y$-intercept | (0, c) | $(0, ?)$ <br> Plug $x=0$ into the equation to find $y$ |
| point symmetric to $y$-intercept | The $y$-intercept (and other points) can be reflected across the axis of symmetry to find other points on the graph of the function. |  |
| $x$-intercept(s) | These points can be read from the graph or table. When in doubt, use the calculator's Calc 2: zero command ( $2^{\text {nd }}$, TRACE $)$. |  |

The two forms of quadratic equations provide information about the function's graph in different ways.

However, some things are the same, regardless of which form you use.

Find each characteristic for the functions described.

| Characteristic | $y=x^{2}-2 x-3$ | $y=(x-1)^{2}-4$ |
| :--- | :--- | :--- |
| vertex |  |  |
| axis of <br> symmetry |  |  |
| $y$-intercept |  |  |
| symmetric <br> point to <br> $y$-intercept |  |  |
| x-intercept(s) |  |  |

Compare with the results from the handout: Investigating Characteristics of Quadratic Functions.

## Characteristics of Quadratic Functions (pp. 2 of 5 )

## Sample Problems

Find the characteristic parts of each function. Use this information to produce the graph.
A) $y=-x^{2}+6 x-2$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



B) $\quad f(x)=2(x+1)^{2}+3$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :--- | :--- |
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|  |  |
|  |  |
|  |  |
|  |  |

## Characteristics of Quadratic Functions (pp. 3 of 5 )

## Practice Problems

For problems \#1-4 make a table of values, graph the function, find the vertex, determine if the vertex is a maximum or minimum, write the equation of the line for the axis of symmetry, find the $y$-intercept and symmetric point, and give the $x$-intercepts.

1) $f(x)=x^{2}+4 x-5$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



2) $y=(x-2)^{2}$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :--- | :--- |
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|  |  |
|  |  |

## Characteristics of Quadratic Functions (pp. 4 of 5)

3) $y=-x^{2}-4 x+12$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



4) $\quad y=2(x+1)^{2}+1$

| Characteristic | Value |
| :---: | :---: |
| Vertex |  |
| Axis of <br> Symmetry |  |
| $y$-intercept |  |
| Point <br> symmetric to <br> $y$-intercept |  |
| $x$-intercept(s) |  |



| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
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|  |  |
|  |  |

## Characteristics of Quadratic Functions (pp. 5 of 5 )


5) True Value Fabricators produces circular iron cast disks to be used as endplates for pipes. The cost of the disks is a quadratic function of the diameter. The cost of some disks is given at right.

1 inch diameter .... \$12.00
2 inch diameter .... $\$ 18.00$
3 inch diameter .... \$28.00
4 inch diameter .... \$42.00
5 inch diameter .... \$60.00
A) In this situation, what are the independent and dependent variables?
B) Sketch a scatterplot of the data. Label the axes.
C) Enter the data into the graphing calculator. Use transformations of $y=x^{2}$ to determine a representative function for the data set in $y=a(x-h)^{2}+k$ form.
D) What would be a reasonable domain and range for this function?

E) Find each of the characteristics and explain their meaning in the problem situation.

| Characteristic | Value(s) | Meaning in Problem Situation |
| :--- | :--- | :--- |
| vertex |  |  |
| axis of <br> symmetry |  |  |
| $y$-intercept |  |  |
| x-intercept(s) |  |  |

F) What would be the cost of a disk with a diameter of 12 inches?
G) If the cost of the disk is $\$ 522$, what would be the diameter of the disk?

