Discontinuities in the Graphs of Piece-wise Defined Functions

The goal of today's lesson is to predict, without graphing, the behavior of the graphs of the pieces in relation to each other. There are three different scenarios that can occur. Draw an example of each.

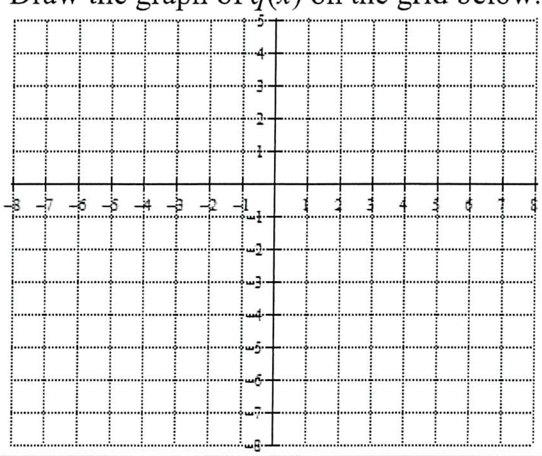
The pieces can connect at a closed circle at $x = a$	The pieces can connect at an open circle at $x = a$	The pieces cannot connect with each other at all at $x = a$

The latter two of these three scenarios are examples of functions that have discontinuity. Before we take a look at graphs that have discontinuities, let's investigate one function in which there is no discontinuity. That is, a function whose pieces connect at a closed circle.

$$q(x) = \begin{cases} -(x+3)^2 + 4, & -5 \le x < 0 \\ |x-2| - 7, & x \ge 0 \end{cases}$$

The first piece of the graph of $q(x)$ will end just shy of $x =$ _____ and the second piece of $q(x)$ will begin at $x =$ _____.

Draw the graph of $q(x)$ on the grid below.



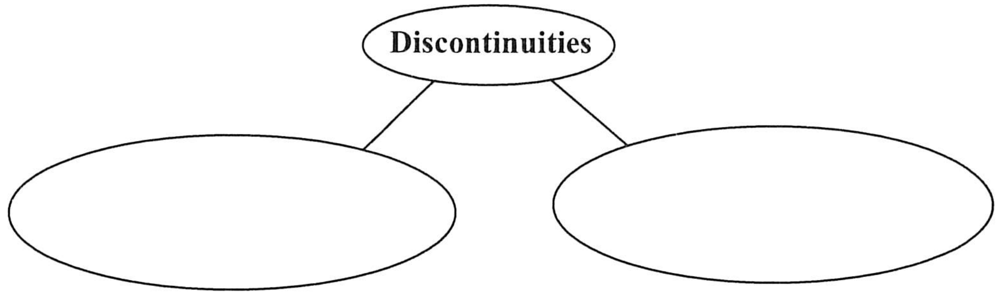
Is $q(x)$ defined at the x – value where the first piece ends and the second piece begins? If so, find the value of y at that x – value.

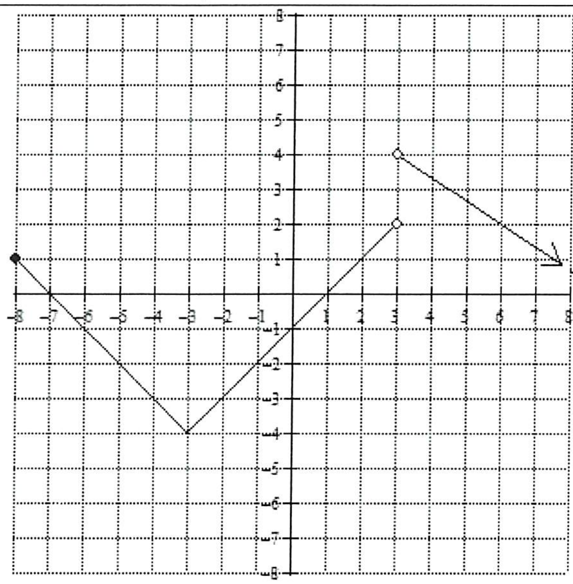
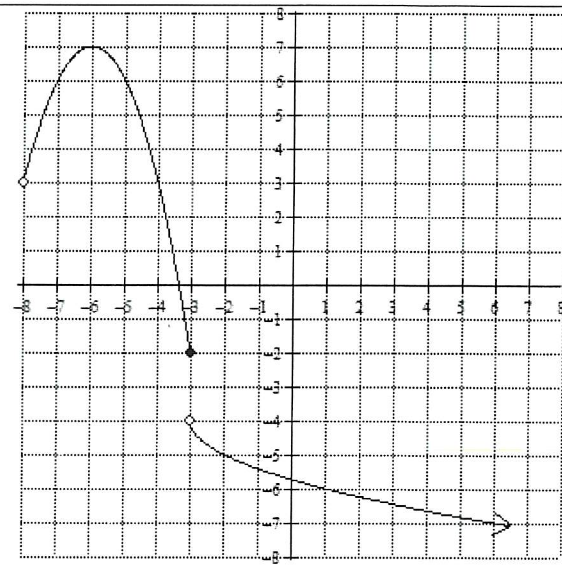
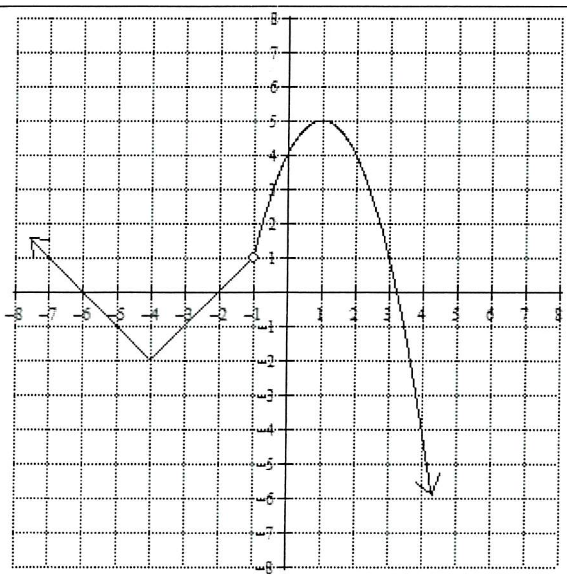
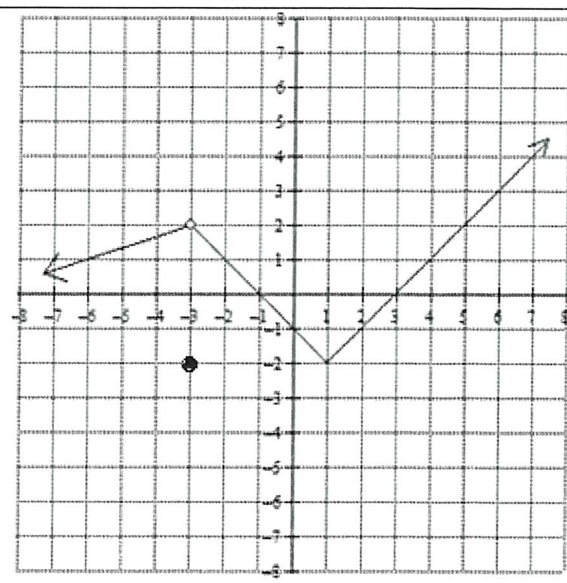
Complete the following statements according to the graph.

As $x \to 0^-$, the graph of $q(x) \to$ __________.

As $x \to 0^+$, the graph of $q(x) \to$ __________.

Using the equation of $q(x)$, determine the y – values that each piece approaches from the left and the right of $x = 0$.



$f(x)=\begin{cases} \lvert x+3\rvert-4, & -8\le x<3 \\ -\frac{1}{3}x+5, & x>3 \end{cases}$	$g(x)=\begin{cases} -(x+6)^2+7, & -8<x\le -3 \\ -\sqrt{x+3}-4, & x>-3 \end{cases}$
	
$h(x)=\begin{cases} \lvert x+4\rvert-2, & x<-1 \\ -(x-1)^2+5, & x>-1 \end{cases}$	$p(x)=\begin{cases} \frac{1}{3}x+3, & x<-3 \\ -2, & x=-3 \\ \lvert x-1\rvert-2, & x>-3 \end{cases}$
	

Complete the following tables of information using the graphs of $f(x)$, $g(x)$, $h(x)$ and $p(x)$.

<table>
<tr><th colspan="2">$f(x)$</th></tr>
<tr><td>What type of discontinuity occurs in the graph of $f(x)$ and at what x – value does it occur?</td><td>Is $f(x)$ defined at this x value? If so, what is the y – value?</td></tr>
<tr><td>By investigating the constraints of the equation, the first piece of the graph of $f(x)$ will stop just shy of $x =$ _____ and the second piece of $f(x)$ will begin just past $x =$ _____.</td><td>Complete the following statements according to the graph.
As $x \to 3^-$, the graph of $f(x) \to$ __________.
As $x \to 3^+$, the graph of $f(x) \to$ __________.</td></tr>
<tr><td colspan="2">Using the equation of $f(x)$, determine the y – values that each piece approaches from the left and the right of $x = 3$.</td></tr>
</table>

<table>
<tr><th colspan="2">$g(x)$</th></tr>
<tr><td>What type of discontinuity occurs in the graph of $g(x)$ and at what x – value does it occur?</td><td>Is $g(x)$ defined at this x value? If so, what is the y – value?</td></tr>
<tr><td>By investigating the constraints of the equation, the first piece of the graph of $g(x)$ will stop at $x =$ _____ and the second piece of $g(x)$ will begin just past $x =$ _____.</td><td>Complete the following statements according to the graph.
As $x \to -3^-$, the graph of $g(x) \to$ __________.
As $x \to -3^+$, the graph of $g(x) \to$ __________.</td></tr>
<tr><td colspan="2">Using the equation of $g(x)$, determine the y – values that each piece approaches from the left and the right of $x = -3$.</td></tr>
</table>

***h*(*x*)**	
What type of discontinuity occurs in the graph of $h(x)$ and at what x – value does it occur?	Is $h(x)$ defined at this x value? If so, what is the y – value?
By investigating the constraints of the equation, the first piece of the graph of $h(x)$ will stop just shy of $x =$ _____ and the second piece of $h(x)$ will begin just past $x =$ _____.	Complete the following statements according to the graph. As $x \to -1^-$, the graph of $h(x) \to$ __________. As $x \to -1^+$, the graph of $h(x) \to$ __________.
Using the equation of $h(x)$, determine the y – values that each piece approaches from the left and the right of $x = -1$.	

***p*(*x*)**	
What type of discontinuity occurs in the graph of $p(x)$ and at what x – value does it occur?	Is $p(x)$ defined at this x value? If so, what is the y – value?
By investigating the constraints of the equation, the first piece of the graph of $p(x)$ will stop just shy of $x =$ _____ and the second piece of $p(x)$ will begin just past $x =$ _____.	Complete the following statements according to the graph. As $x \to -3^-$, the graph of $p(x) \to$ __________. As $x \to -3^+$, the graph of $p(x) \to$ __________.
Using the equation of $p(x)$, determine the y – values that each piece approaches from the left and the right of $x = -3$.	

After investigating the graphs and equations of $q(x)$, $f(x)$, $g(x)$, $h(x)$ and $p(x)$ by answering the questions about each, what are some general conclusions that you can make about when a function will be continuous at $x = a$ and when there will be point or jump discontinuities at $x = a$.

Continuity at $x = a$	
Jump Discontinuity at $x = a$	Point Discontinuity at $x = a$

For each of the functions below, determine the values at which the graph could be discontinuous. Then, determine if the function is actually discontinuous at those values. If the function is discontinuous, then classify the discontinuity as either jump or point discontinuities. Show your work and explain your reasoning for each.

1. $g(x) = \begin{cases} -(x+4)^2 + 3, & x < -1 \\ |x-3| - 10, & x > -1 \end{cases}$

2. $g(x) = \begin{cases} -2|x+3| + 2, & -6 < x < -1 \\ |x+3| - 4, & x \geq -1 \end{cases}$

3. $h(x)=\begin{cases} 2x-3, & x<3 \\ x^2-2x, & 3\le x<5 \\ |x-3|+2, & x\ge 5 \end{cases}$

For what value(s) of a and b will the function below have a point discontinuities at $x=-3$ and $x=2$?

$$f(x)=\begin{cases} 2x+3, & x<-3 \\ ax+3b, & -3<x<2 \\ \frac{1}{2}x^2+3x-1, & x>2 \end{cases}$$